

## QUANTITATIVE METHODS

### TIME VALUE OF MONEY BASICS

**Future value (FV):** amount to which investment grows after one or more compounding periods.

- **Future value:**  $FV = PV(1 + I/Y)^N$ .
- **Present value (PV):** current value of some future cash flow  $PV = FV/(1 + I/Y)^N$ .
- **Annuities:** series of equal cash flows that occur at evenly spaced intervals over time.
- **Ordinary annuity:** cash flow at end-of-time period.
- **Annuity due:** cash flow at beginning-of-time period.
- **Perpetuities:** annuities with infinite lives.  
 $PV_{\text{perpetuity}} = PMT/(\text{discount rate})$ .

### REQUIRED RATE OF RETURN

**Components:**

1. Real risk-free rate (RFR).
2. Expected inflation rate premium (IP).
3. Risk premium.

$$E(R) = (1 + R_{R_{\text{real}}})(1 + IP)(1 + RP) - 1$$

Approximation formula for nominal required rate:

$$E(R) \cong RFR + IP + RP$$

**Means**

**Arithmetic mean:** sum of all observation values in sample/population, divided by # of observations.

Geometric mean: used when calculating investment returns over multiple periods or to measure compound growth rates.

**Geometric mean return:**

$$R_G = [(1 + R_1) \times \dots \times (1 + R_N)]^{1/N} - 1$$

**Harmonic mean =** 
$$\bar{X}_H = \frac{N}{\sum_{i=1}^N \frac{1}{X_i}}$$

**Trimmed mean (x%):** Exclude highest and lowest x/2 percent of observations.

**Winsorized mean (x%):** Substitute values for highest and lowest x/2 percent of observations.

### VARIANCE AND STANDARD DEVIATION

**Variance:** average of squared deviations from mean.

**sample variance =** 
$$s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$$

**Standard deviation:** square root of variance.

### TARGET DOWNSIDE DEVIATION

$$S_{\text{target}} = \sqrt{\frac{\sum_{\text{all } X_i < \text{target}} (X_i - \text{target})^2}{n - 1}}$$

### HOLDING PERIOD RETURN (HPR)

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \text{ or } \frac{P_t + D_t}{P_{t-1}} - 1$$

### COEFFICIENT OF VARIATION

Coefficient of variation (CV): expresses how much dispersion exists relative to mean of a distribution; allows for direct comparison of dispersion across different data sets. CV is calculated by dividing standard deviation of a distribution by the mean or expected value of the distribution:

$$CV = \frac{s}{\bar{X}}$$

### ROY'S SAFETY-FIRST RATIO

$$\frac{\bar{r}_p - r_{\text{target}}}{\sigma_p}$$

### EXPECTED RETURN/STANDARD DEVIATION

**Expected return-**  $E(X) = \sum P(x_i)x_n$

$$E(X) = P(x_1)x_1 + P(x_2)x_2 + \dots + P(x_n)x_n$$

**Probabilistic variance-**  $\sigma^2(X) = \sum P(x_i)[x_i - E(X)]^2$

$$= P(x_1)[x_1 - E(X)]^2 + P(x_2)[x_2 - E(X)]^2 + \dots + P(x_n)[x_n - E(X)]^2$$

**Standard deviation:** take square root of variance.

### CORRELATION AND COVARIANCE

**Correlation:** covariance divided by product of the two standard deviations.

$$\text{Corr}(R_i, R_j) = \frac{\text{Cov}(R_i, R_j)}{\sigma(R_i)\sigma(R_j)}$$

**Expected return, variance of 2-stock portfolio:**

$$E(R_p) = w_A E(R_A) + w_B E(R_B)$$

$$\text{var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B \sigma(R_A)\sigma(R_B)\rho(R_A, R_B)$$

### NORMAL DISTRIBUTIONS

Normal distribution is completely described by its mean and variance.

68% of observations fall within  $\pm 1\sigma$ .

90% fall within  $\pm 1.65\sigma$ .

95% fall within  $\pm 1.96\sigma$ .

99% fall within  $\pm 2.58\sigma$ .

### COMPUTING Z-SCORES

Z-score: "standardizes" observation from normal distribution; represents # of standard deviations a given observation is from population mean.

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

## BINOMIAL MODELS

**Binomial distribution:** assumes a variable can take one of two values (success/failure) or, in the case of a stock, movements (up/down). A binomial model can describe changes in the value of an asset or portfolio; it can be used to compute its expected value over several periods.

## SAMPLING DISTRIBUTION

**Sampling distribution:** probability distribution of all possible sample statistics computed from a set of equal-size samples randomly drawn from the same population. The sampling distribution of the mean is the distribution of estimates of the mean.

## CENTRAL LIMIT THEOREM

**Central limit theorem:** when selecting simple random samples of size  $n$  from population with mean  $\mu$  and finite variance  $\sigma^2$ , the sampling distribution of sample mean approaches normal probability distribution with mean  $\mu$  and variance equal to  $\sigma^2/n$  as the sample size becomes large.

## STANDARD ERROR

Standard error of the sample mean is the standard deviation of distribution of the sample means.

known population variance:  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

unknown population variance:  $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

## CONFIDENCE INTERVALS

Confidence interval: gives range of values the mean value will be between, with a given probability (say 90% or 95%). With known variance, formula for a confidence interval is:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$z_{\alpha/2} = 1.645$  for 90% confidence intervals  
(significance level 10%, 5% in each tail)

$z_{\alpha/2} = 1.960$  for 95% confidence intervals  
(significance level 5%, 2.5% in each tail)

$z_{\alpha/2} = 2.575$  for 99% confidence intervals  
(significance level 1%, 0.5% in each tail)

## NULL AND ALTERNATIVE HYPOTHESES

**Null hypothesis ( $H_0$ ):** hypothesis that contains the equal sign ( $=, \leq, \geq$ ).

**Alternative hypothesis ( $H_a$ ):** concluded if there is sufficient evidence to reject the null hypothesis.

## DIFFERENCE BETWEEN ONE- AND TWO-TAILED TESTS

**One-tailed test:** tests whether value is greater than or less than a given number.

**Two-tailed test:** tests whether value is equal to a given number.

## TYPE I AND TYPE II ERRORS

- Type I error: rejection of null hypothesis when it is actually true.
- Type II error: failure to reject null hypothesis when it is actually false.

## HYPOTHESIS TESTS

Test of:	Stat	d.f.
Mean	t or z	n - 1
Difference in means	t	n - 1
Mean differences	t	n - 1
Variance	$\chi^2$	n - 1
Equal variances	F	$n_1 - 1, n_2 - 1$
Correlation	t	n - 2
Independence	$\chi^2$	$(r - 1)(c - 1)$
Regression slope: Significance Value	F t	1, n - 2 n - 2

## ECONOMICS

### ELASTICITY

$$\text{Own price elasticity} = \frac{\% \Delta \text{quantity demanded}}{\% \Delta \text{price}}$$

If absolute value  $> 1$ , demand is elastic.

If absolute value  $< 1$ , demand is inelastic.

On a straight line demand curve, total revenue is maximized where price elasticity =  $-1$ .

$$\text{Income elasticity} = \frac{\% \Delta \text{quantity demanded}}{\% \Delta \text{income}}$$

If positive, the good is a normal good.

If negative, the good is an inferior good.

$$\text{Cross price elasticity} = \frac{\% \Delta \text{quantity demanded}}{\% \Delta \text{price of related good}}$$

If positive, related good is a substitute.

If negative, related good is a complement.

### BREAKEVEN AND SHUTDOWN

**Breakeven:** total revenue = total cost.

Operate in short run if total revenue is greater than total variable cost but less than total cost. Shut down in short run if total revenue is less than total variable cost.

## MARKET STRUCTURES

**Perfect competition:** Many firms with no pricing power; very low or no barriers to entry; homogeneous product.

**Monopolistic competition:** Many firms; some pricing power; low barriers to entry; differentiated products; large advertising expense.

**Oligopoly:** Few firms that may have significant pricing power; high barriers to entry; products may be homogeneous or differentiated.

**Monopoly:** Single firm with significant pricing power; high barriers to entry; advertising used to compete with substitute products. In all market structures, profit is maximized at the output quantity for which marginal revenue = marginal cost.

## GROSS DOMESTIC PRODUCT

Real GDP = consumption spending + investment + government spending + net exports.

## SAVINGS, INVESTMENT, FISCAL BALANCE, AND TRADE BALANCE

Fiscal budget deficit ( $G - T$ ) = excess of saving over domestic investment ( $S - I$ ) - trade balance ( $X - M$ )

## EQUATION OF EXCHANGE

$MV = PY$ , where  $M$  = real money supply,  $V$  = velocity of money in transactions,  $P$  = price level, and  $Y$  = real GDP

## BUSINESS CYCLE PHASES

Expansion; peak; contraction; trough.

## ECONOMIC INDICATORS

**Leading:** Turning points occur ahead of peaks and troughs (stock prices, initial unemployment claims, manufacturing new orders)

**Coincident:** Turning points coincide with peaks and troughs (nonfarm payrolls, personal income, manufacturing sales)

**Lagging:** Turning points follow peaks and troughs (average duration of unemployment, inventory/ sales ratio, prime rate)

## FACTORS AFFECTING AGGREGATE DEMAND

Consumers' wealth; business expectations; consumers' income expectations; capacity utilization; monetary and fiscal policy; exchange rates; global economic growth.

## FACTORS AFFECTING SR AGGREGATE SUPPLY

Input prices; labor productivity; expectations for output prices; taxes and subsidies; exchange rates; all factors that affect LR aggregate supply.

## FACTORS AFFECTING LR AGGREGATE SUPPLY

Size of labor force; human capital; supply of natural resources; stock of physical capital; level of technology.

## TYPES OF UNEMPLOYMENT

**Frictional:** time lag in matching qualified workers with job openings.

**Structural:** unemployed workers do not have the skills to match newly created jobs.

**Cyclical:** economy producing at less than capacity during contraction phase of business cycle.

## POLICY MULTIPLIERS

$$\text{money multiplier} = \frac{1}{\text{reserve requirement}}$$

$$\text{money multiplier} = \frac{1}{1 - \text{MPC}(1 - t)}$$

where MPC = marginal propensity to consume,  $t$  = tax rate.

## EXPANSIONARY AND CONTRACTIONARY POLICY

**Monetary policy** is expansionary when the policy rate is less than the neutral interest rate (real trend rate of economic growth + inflation target) and contractionary when the policy rate is greater than the neutral interest rate.

**Fiscal policy** is expansionary when a budget deficit is increasing or surplus is decreasing, and contractionary when a budget deficit is decreasing or surplus is increasing.

## BALANCE OF PAYMENTS

**Current account:** merchandise and services; income receipts; unilateral transfers.

**Capital account:** capital transfers; sales/purchases of nonfinancial assets.

**Financial account:** government-owned assets abroad; foreign-owned assets in the country.

## REGIONAL TRADING AGREEMENTS

**Free trade area:** Removes barriers to goods and services trade among members.

**Customs union:** Members also adopt common trade policies with non-members.

**Common market:** Members also remove barriers to labor and capital movements among members.

**Economic union:** Members also establish common institutions and economic policy.

**Monetary union:** Members also adopt a common currency.

## FOREIGN EXCHANGE RATES

For the exam, FX rates are expressed as price currency / base currency and interpreted as the number of units of the price currency for each unit of the base currency.

## REAL EXCHANGE RATE

$$\text{nominal FX rate} \times \left( \frac{\text{base currency CPI}}{\text{price currency CPI}} \right)$$

## NO-ARBITRAGE FORWARD EXCHANGE RATE

$$\frac{\text{forward}}{\text{spot}} = \frac{1 + \text{price currency interest rate}}{1 + \text{base currency interest rate}}$$

## EXCHANGE RATE REGIMES

**Formal dollarization:** country adopts foreign currency.

**Monetary union:** members adopt common currency.

**Fixed peg:**  $\pm 1\%$  margin versus foreign currency or basket of currencies.

**Target zone:** Wider margin than fixed peg.

**Crawling peg:** Pegged exchange rate adjusted periodically.

**Crawling bands:** Width of margin increases over time.

**Managed floating:** Monetary authority acts to influence exchange rate but does not set a target.

**Independently floating:** Exchange rate is market determined.

## FINANCIAL STATEMENT ANALYSIS

### REVENUE RECOGNITION

Two requirements: (1) completion of earnings process and (2) reasonable assurance of payment.

#### Five-step revenue recognition model:

1. Identify contracts
2. Identify performance obligations
3. Determine transaction price
4. Allocate price to obligations
5. Recognize when (as) obligations are satisfied

### UNUSUAL OR INFREQUENT ITEMS

- Gains/losses from disposal of a business segment.
- Gains/losses from sale of assets or investments in subsidiaries.
- Provisions for environmental remediation.
- Impairments, write-offs, write-downs, and restructuring costs.
- Integration expenses associated with businesses recently acquired.

### DISCONTINUED OPERATIONS

To be accounted for as a discontinued operation, a business—assets, operations, investing, financing activities—must be physically/operationally distinct from rest of firm. Income/losses are reported net of tax after net income from continuing operations.

### COMPUTE CASH FLOWS FROM OPERATIONS (CFO)

**Direct method:** start with cash collections (cash equivalent of sales); cash inputs (cash equivalent of cost of goods sold); cash operating expenses; cash interest expense; cash taxes.

**Indirect method:** start with net income, subtracting back gains and adding back losses resulting from financing or investment cash flows, adding back all noncash charges, and adding and subtracting asset and liability accounts that result from operations.

### FREE CASH FLOW

Free cash flow (FCF) measures cash available for discretionary purposes. It is equal to operating cash flow less net capital expenditures.

### CRITICAL RATIOS

#### Common-size financial statement analysis:

- Common-size balance sheet expresses all balance sheet accounts as a percentage of total assets.

- Common-size income statement expresses all income statement items as a percentage of sales.
- Common-size cash flow statement expresses each line item as a percentage of total cash inflows (outflows), or as a percentage of net revenue.

#### Horizontal common-size financial statement analysis:

expresses each line item relative to its value in a common base period.

$$\text{Liquidity ratios: current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{quick ratio} = \frac{\text{cash} + \text{marketable securities} + \text{receivables}}{\text{current liabilities}}$$

$$\text{cash ratio} = \frac{\text{cash} + \text{marketable securities}}{\text{current liabilities}}$$

$$\text{defensive interval} = \frac{\text{cash} + \text{mkt. sec.} + \text{receivables}}{\text{daily cash expenditures}}$$

#### Receivables, inventory, payables turnover, and days supply ratios—all of which are used in the cash conversion cycle:

$$\text{receivables turnover} = \frac{\text{annual sales}}{\text{average receivables}}$$

$$\text{inventory turnover} = \frac{\text{cost of goods sold}}{\text{average inventory}}$$

$$\text{payables turnover ratio} = \frac{\text{purchases}}{\text{average trade payables}}$$

$$\text{days of sales outstanding} = \frac{365}{\text{receivables turnover}}$$

$$\text{days of inventory onhand} = \frac{365}{\text{inventory turnover}}$$

$$\text{number of days of payables} = \frac{365}{\text{payables turnover ratio}}$$

$$\text{cash conversion cycle} = \left( \frac{\text{days of inventory onhand}}{\text{inventory turnover}} \right) + \left( \frac{\text{days of sales outstanding}}{\text{receivables turnover}} \right) - \left( \frac{\text{number of days of payables}}{\text{payables turnover ratio}} \right)$$

#### Total asset, fixed-asset, and working capital turnover ratios:

$$\text{total asset turnover} = \frac{\text{revenue}}{\text{average total assets}}$$

$$\text{fixed asset turnover} = \frac{\text{revenue}}{\text{average fixed assets}}$$

$$\text{working capital turnover} = \frac{\text{revenue}}{\text{average working capital}}$$

### Gross, operating, and net profit margins:

$$\text{gross profit margin} = \frac{\text{gross profit}}{\text{revenue}}$$

$$\text{operating profit margin} = \frac{\text{operating profit}}{\text{revenue}} = \frac{\text{EBIT}}{\text{net sales}}$$

$$\text{net profit margin} = \frac{\text{net income}}{\text{revenue}}$$

### Return on assets [return on total capital (ROTC)]:

$$\text{return on assets (total capital)} = \frac{\text{EBIT}}{\text{average total capital}}$$

### Debt to equity ratio and total debt ratio:

$$\text{debt-to-equity ratio} = \frac{\text{total debt}}{\text{total equity}}$$

$$\text{total-debt-ratio} = \frac{\text{total debt}}{\text{total assets}}$$

### Interest coverage and fixed charge coverage:

$$\text{interest coverage} = \frac{\text{EBIT}}{\text{interest}}$$

$$\text{fixed charge coverage} = \frac{\text{EBIT} + \text{lease payments}}{\text{interest} + \text{lease payments}}$$

### Growth rate (g): $g = RR \times ROE$

$$\text{retention rate} = 1 - \frac{\text{dividends declared}}{\text{operating income after taxes}}$$

**Liquidity ratios** indicate company's ability to pay its short-term liabilities.

**Operating performance ratios** indicate how well management operates the business.

### DuPONT ANALYSIS

#### Traditional DuPont equation:

$$\text{return on equity} = \left( \frac{\text{net income}}{\text{sales}} \right) \left( \frac{\text{sales}}{\text{assets}} \right) \left( \frac{\text{assets}}{\text{equity}} \right)$$

You may also see it presented as:

$$\text{return on equity} = \left( \frac{\text{net profit}}{\text{margin}} \right) \left( \frac{\text{asset}}{\text{turnover}} \right) \left( \frac{\text{equity}}{\text{multiplier}} \right)$$

**Extended DuPont equation** further decomposes net profit margin:

$$\text{ROE} = \left( \frac{\text{net income}}{\text{EBT}} \right) \times \left( \frac{\text{EBT}}{\text{EBIT}} \right) \times \left( \frac{\text{EBIT}}{\text{revenue}} \right) \times \left( \frac{\text{revenue}}{\text{avg. total assets}} \right) \times \left( \frac{\text{avg. total assets}}{\text{avg. equity}} \right)$$

You may also see it presented as:

$$\text{ROE} = \text{tax burden} \times \text{interest burden} \times \text{EBIT margin} \times \text{asset turnover} \times \text{leverage}$$

### MARKETABLE SECURITY CLASSIFICATIONS

**Held-for-trading:** fair value on balance sheet; dividends, interest, realized and unrealized G/L recognized on income statement.

**Available-for-sale:** fair value on balance sheet; dividends, interest, realized G/L recognized on income statement; unrealized G/L is other comprehensive income.

**Held-to-maturity:** amortized cost on balance sheet; interest, realized G/L recognized on income statement.

### INVENTORY ACCOUNTING

In periods of rising prices and stable or increasing inventory quantities:

#### LIFO results in:

Higher COGS  
Lower gross profit  
Lower inventory balances

#### FIFO results in:

Lower COGS  
Higher gross profit  
Higher inventory balances

**Basic EPS** calculation **does not** consider effects of any dilutive securities in computation of EPS:

$$\text{basic EPS} = \frac{\text{net income} - \text{preferred dividends}}{\text{wtd. avg. no. of common shs. outstanding}}$$

$$\text{diluted EPS} = \frac{\text{adj. income avail. for common shares}}{\text{wtd. avg. common shares plus potential common shares outstanding}}$$

Therefore, diluted EPS is:

$$\frac{|\text{net income} - \text{pfd div}| + \left( \frac{\text{convertible preferred dividends}}{\text{wtd avg sh's}} \right) + \left( \frac{\text{convertible debt interest}}{\text{sh's from conversion of conv. debt}} \right) + \left( \frac{\text{share issuable from stock options}}{\text{share}} \right) (1 - t)}{\left( \frac{\text{wtd avg sh's}}{\text{wtd avg sh's}} \right) + \left( \frac{\text{shares from conversion of conv. pfd. sh's}}{\text{conv. pfd. sh's}} \right) + \left( \frac{\text{sh's from conversion of conv. debt}}{\text{conv. debt}} \right) + \left( \frac{\text{share issuable from stock options}}{\text{share}} \right)}$$

### LONG-LIVED ASSETS CAPITALIZING VS. EXPENSING

**Capitalizing:** lowers income variability and increases near-term profits. Increase assets, equity.

**Expensing:** opposite effect.

### DEPRECIATION

$$\text{Straight-line} = \frac{\text{cost} - \text{residual value}}{\text{useful life}}$$

#### Double declining balance:

$$\left( \frac{2}{\text{useful life}} \right) (\text{cost} - \text{accum. depreciation})$$

Units of production:

$$\frac{\text{cost} - \text{salvage value}}{\text{useful life in units}} \times \text{output units}$$

## REVALUATION OF LONG-LIVED ASSETS

**IFRS:** revaluation gain recognized in net income only to the extent it reverses previously recognized impairment loss; further gains recognized in equity as revaluation surplus. (For investment property, all gains and losses from marking to fair value are recognized as income.)

**U.S. GAAP:** revaluation is not permitted.

## DEFERRED TAXES

Created when taxable income (on tax return) ≠ pretax income (on financial statements) due to temporary differences.

**Deferred tax liabilities** are created when taxable income < pretax income. Treat DTL as equity if not expected to reverse.

**Deferred tax assets** are created when taxable income > pretax income. Must recognize **valuation allowance** if more likely than not that DTA will not be realized.

## LONG-TERM LIABILITIES

**Premium bond:** coupon rate > market rate at issuance.

**Discount bond:** coupon rate < market rate at issuance.

**Interest expense:** equals book value at the beginning of the year multiplied by the market rate of interest at the time the bonds were issued.

## LEASES

**Lessee reporting:** Under IFRS, lessee recognizes asset and liability equal to PV of lease payments. Interest portion of each payment is interest expense, principal portion decreases liability. U.S. GAAP is same except that for an operating lease, entire lease payment is an expense.

**Lessor reporting, finance lease:** Remove asset from balance sheet, recognize lease receivable asset, report interest income.

**Lessor reporting, operating lease:** Keep asset on balance sheet, report lease payments as income, record depreciation expense.

## PENSIONS

**Defined contribution:** employer contribution expensed in period incurred.

**Defined benefit:** overfunded plan recognized as asset, underfunded plan recognized as liability.

## CORPORATE ISSUERS

### WEIGHTED AVERAGE COST OF CAPITAL

$$WACC = (w_d)[k_d(1-t)] + (w_{ps})(k_{ps}) + (w_{ce})(k_s)$$

### COST OF PREFERRED STOCK

$$k_p = \frac{D_{ps}}{P}$$

### COST OF EQUITY USING CAPM

$$k_e = RFR + \beta(R_{mkt} - RFR)$$

## CAPITAL ALLOCATION

$$NPV = CF_0 + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \dots + \frac{CF_n}{(1+k)^n}$$

**IRR:** discount rate that makes NPV equal to zero.

## NONPUBLIC/THINLY TRADED STOCK BETA

Delevered asset beta for comparable company:

$$\beta_{asset} = \beta_{equity} \times \frac{1}{1 + \left( (1-t) \frac{D}{E} \right)}$$

Relevered beta for target company:

$$\beta_{target} = \beta_{asset} \times \left[ 1 + \left( (1-t) \frac{D}{E} \right) \right]$$

Adjusted beta = 2/3 x unadjusted beta + 1/3

## MEASURES OF LEVERAGE

**Total leverage:** percent change in net income from a given percent change in sales.

**Operating leverage:** percent change in EBIT a given percent change in sales.

**Financial leverage:** percent change in net income from a given percent change in EBIT.

breakeven quantity of sales =

$$\frac{\text{fixed operating \& financing costs}}{\text{price} - \text{variable costs per unit}}$$

operating breakeven quantity of sales =

$$\frac{\text{fixed operating costs}}{\text{price} - \text{variable costs per unit}}$$

## CORPORATE GOVERNANCE

**Stakeholder groups:** Shareholders, board of directors, senior managers, employees, creditors, suppliers.

**Board committees:** Audit, governance, nominations, compensation, risk, investment.

## ESG IN INVESTMENT ANALYSIS

**Negative screening:** Exclude companies with poor ESG records.

**Positive screening:** Invest in companies with good ESG records.

**Thematic investing:** Invest to promote a specific ESG goal.

**Active ownership:** Use share voting, access to managers to promote ESG goals.

## SECURITIES MARKETS & EQUITY INVESTMENTS

### WELL-FUNCTIONING SECURITY MARKETS

- Operational efficiency (lowest possible transactions costs).

Informational efficiency (prices rapidly adjust to new information).

## MARGIN PURCHASES

**For margin transactions:**

- Leverage factor = 1/margin percentage.
- Levered return = HPR x leverage factor.

## MARGIN CALL PRICE

$$\frac{P_0(1 - \text{initial margin}\%)}{1 - \text{maintenance margin}\%}$$

## COMPUTING INDEX PRICES

$$\text{Price-weighted Index} = \frac{\sum \text{stock prices}}{\text{adjusted divisor}}$$

Value-weighted Index

$$= \frac{\sum (\text{current prices})(\# \text{ shares})}{\sum (\text{base year prices})(\# \text{ base year shares})} \times \text{base value}$$

## TYPES OF ORDERS

**Execution instructions:** how to trade; e.g., market orders, limit orders.

**Validity instructions:** when to execute; e.g., stop orders, day orders, fill-or-kill orders.

**Clearing instructions:** how to clear and settle; for sell orders, specify short sale or sale of owned security.

## MARKET STRUCTURES

**Quote-driven markets:** investors trade with dealers.

**Order-driven markets:** buyers and sellers matched by rules.

**Brokered markets:** brokers find counterparties.

## FORMS OF EMH

**Weak form.** Current stock prices **fully reflect available security market info.** Volume information/past price do not relate to future direction of security prices. Investor cannot achieve excess returns using tech analysis.

**Semi-strong form.** Security prices instantly adjust to new public information. Investor cannot achieve excess returns using fundamental analysis.

**Strong form.** Stock prices fully reflect all information from public and private sources. Assumes perfect markets in which all information is cost free and available to everyone at the same time. Even with inside info, investor cannot achieve excess returns.

## EQUITY INVESTMENTS

### INDUSTRY LIFE CYCLE STAGES

**Embryonic:** slow growth, high prices, large investment needed, high risk of failure.

**Growth:** rapid growth, falling prices, limited competition, increasing profitability.

**Shakeout:** slower growth, intense competition, declining profitability, cost cutting, weaker firms fail or merge.

**Mature:** slow growth, consolidation, stable prices, high barriers to entry.

**Decline:** negative growth, declining prices, consolidation.

## FIVE COMPETITIVE FORCES

1. Rivalry among existing competitors.
2. Threat of entry.
3. Threat of substitutes.
4. Power of buyers.
5. Power of suppliers.

## ONE-PERIOD VALUATION MODEL

$$V_0 = \frac{D_1}{(1+k_e)} + \frac{P_1}{(1+k_e)}$$

Be sure to use expected dividend  $D_1$  in calculation.

## INFINITE PERIOD DIVIDEND DISCOUNT MODELS

**Supernormal growth model (multi-stage) DDM:**

$$V_0 = \frac{D_1}{(1+k_e)} + \dots + \frac{D_n}{(1+k_e)^n} + \frac{P_n}{(1+k_e)^n}$$

$$\text{where: } P_n = \frac{D_{n+1}}{(k_e + g_c)}$$

**Constant growth model:**

$$V_0 = \frac{D_0(1+g_c)}{k_e - g_c} = \frac{D_1}{k_e - g_c}$$

**Critical relationship between  $k_e$  and  $g_c$ :**

- As difference between  $k_e$  and  $g_c$  widens, value of stock falls.
- As difference narrows, value of stock rises.
- Small changes in difference between  $k_e$  and  $g_c$  cause large changes in stock's value.

**Critical assumptions of infinite period DDM:**

- Stock pays dividends; constant growth rate.
- Constant growth rate,  $g_c$ , never changes.
- $k_e$  must be greater than  $g_c$  (or math will not work).

## EARNINGS MULTIPLIER MODEL

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{k - g} = \frac{\text{payout ratio}}{k - g}$$

## PRICE MULTIPLES

$$\text{leading P/E} = \frac{\text{price per share}}{\text{for ecast EPS next 12 mo.}}$$

$$\text{trailing P/E} = \frac{\text{price per share}}{\text{EPS previous 12 mo.}}$$

$$P/B = \frac{\text{price per share}}{\text{book value per share}}$$

$$P/S = \frac{\text{price per share}}{\text{sales per share}}$$

$$P/CF = \frac{\text{price per share}}{\text{cash flow per share}}$$

## FIXED INCOME

### BASIC FEATURES OF BONDS

**Issuer.** Household, nonfinancial corporations, governments, financial institutions.

**Maturity.** Money market (one year or less); capital market (greater than one year).

**Par value.** Bond's principal value (face value).

**Coupon.** Annual percent of par; fixed or floating. Divide by periodicity to get *periodic* rate.

**Currency.** Single, dual, currency option.

**Indenture.** Affirmative and negative covenants.

### PRICE, YIELD, COUPON RELATIONSHIPS

Bond prices and yields are inversely related. Increase in yield decreases price; decrease in yield increases price.

Coupon < yield: Discount to par value.

Coupon > yield: Premium to par value.

**Constant-yield price trajectory:** Price approaches par as bond nears maturity from amortization of discounts and premiums. Capital gains and losses are calculated relative to this trajectory.

### CASH FLOW STRUCTURES

**Bullet:** All principal repaid at maturity.

**Fully amortizing:** Equal periodic payments include both interest and principal.

**Partially amortizing:** Periodic payments include interest and principal, balloon payment at maturity repays remaining principal.

**Sinking fund:** Schedule for early redemption.

**Floating-rate:** Coupon payments based on reference rate plus margin.

### BOND PRICING

There are two equivalent ways to price a bond:

- Constant discount rate applied to all cash flows (YTM) to find PV. This is a bond's **flat price** (does not include accrued interest).
- Discount each cash flow using appropriate spot rate for each. This is a bond's **no-arbitrage price**.

**Full price** includes accrued interest. Government bonds use actual day counts; corporate bonds use 30/360 method.

$$\text{full price} = \text{PV at last coupon date} \times (1 + \text{YTM})^{t/T}$$

$$\text{accrued interest} = \text{coupon payment} \times (t/T)$$

where:

t = days from most recent coupon payment to trade

settlement

T = days in coupon payment period

**Matrix pricing:** For illiquid bonds, use yields of bonds with same credit quality to estimate yield; adjust for maturity differences with linear interpolation.

### BOND MARKETS

**National bond market** includes domestic bonds and foreign bonds.

- **Domestic bonds.** Domestic issuer and currency.
- **Foreign bonds.** Foreign issuer, domestic currency.

**Eurobond market** is outside any one country, with bonds denominated in currencies other than those of countries in which bonds are sold.

**Global bonds** trade in both a national bond market and the eurobond market.

### BOND ISSUANCE

**Underwritten offering:** Investment banks buy entire issue, sell to public.

**Best efforts offering:** Investment banks act as brokers.

**Shelf registration:** Register entire issue with regulators but sell over a period of time.

### EMBEDDED OPTIONS

Callable: Issuer may repay principal early. Increases yield and decreases duration.

Puttable: Bondholder may sell bond back to issuer. Decreases yield and duration.

Convertible: Bondholder may exchange bond for issuer's common stock.

**Embedded warrants:** Bondholder may buy issuer's common stock at exercise price.

### YIELD MEASURES

**Effective yield** depends on periodicity. YTM = effective yield for annual-pay bonds.

**Semiannual bond basis:** YTM = 2 x semiannual discount rate.

**Current yield** = annual coupon / price.

**Simple yield** = current yield ± amortization.

**Yield to call** is based on call date and call price.

**Yield to worst** is lowest of a bond's YTCs or YTM.

**Money market yields** may be on a discount or add-on basis and may use a 360- or 365-day year.

**Bond-equivalent yield** is an annualized add-on yield based on a 365-day year.

### FORWARD AND SPOT RATES

Forward rate is a rate for a loan that begins at a future date. "1y3y" = 3-year forward rate 1 year from today.

**Example of spot-forward relationship:**

$$(1 + S_2)^2 = (1 + S_1)(1 + 1y_1)$$

### YIELD SPREADS

G-spread: Basis points above government yield.

I-spread: Basis points above swap rate.

Z-spread: Accounts for shape of yield curve.

Option-adjusted spread: Adjusts Z-spread for effects of embedded options.



## INTEREST RATE RISK

Interest rate risk has two components: **reinvestment risk** and **market price risk** from YTM changes. These risks have opposing effects on an investor's horizon yield.

- Bond investors with short horizons are more concerned with market price risk.
- Bond investors with long horizons are more concerned with reinvestment risk.
- The horizon at which market price risk and reinvestment risk just offset is a bond's **Macaulay duration**. This is the weighted average of times until a bond's cash flows are scheduled to be paid.

**Modified duration** is the approximate change in a bond's price given a 1% change in its YTM:

$$\frac{\text{Macaulay duration}}{(1+r)} \approx \frac{(V_-) - (V_+)}{2V_0(\Delta y)}$$

**Effective duration** is required if a bond has embedded options:

$$\frac{(V_-) - (V_+)}{2V_0(\Delta \text{curve})}$$

Price change estimates based on duration only are improved by adjusting for **convexity**:

$$\% \Delta \text{price} = -\text{duration}(\Delta y) + \frac{1}{2} \text{convexity}(\Delta y)^2$$

## ASSET-BACKED SECURITIES

**Residential MBS'**, home mortgages are collateral.

**Agency RMBS** include only conforming loans;

**nonagency RMBS** may include nonconforming loans and need credit enhancement.

Prepayment risk: contraction risk from faster prepayments; extension risk from slower prepayments.

**CMOs**: pass-through MBS are collateral. May have sequential -pay or PAC/support structure.

**Commercial MBS**: non-recourse mortgages on commercial properties are collateral.

**Auto ABS**: auto loans are collateral.

**Credit card ABS**: credit card receivables are collateral.

**CDOs**: Bonds, bank loans, MBS, ABS, or other CDOs are collateral.

## COLLATERAL AND CREDIT ENHANCEMENT

**Secured bonds** are backed by specific collateral and senior to unsecured bonds.

**Unsecured bonds** are general claims to issuer's cash flows and assets.

**Internal credit enhancement**: Excess spread, over collateralization, waterfall structure.

**External credit enhancement**: Surety bonds, letters of credit, bank guarantees.

## CREDIT ANALYSIS

**Investment grade**: Baa3/BBB- or above

**Non-investment grade**: Ba1/BB+ or below

**Corporate family rating (CFR)**: issuer rating.

**Corporate credit rating (CCR)**: security rating.

**"Four Cs"**: capacity, collateral, covenants, character, default risk = probability of default

loss severity = percent of value lost if borrower defaults

expected loss = default risk x loss severity

recovery rate = 1 - expected loss percentage

## ALTERNATIVE INVESTMENTS

### HEDGE FUNDS

**Event-driven strategies**: merger arbitrage; distressed/restructuring; activist shareholder; special situations.

**Relative value strategies**: convertible arbitrage; asset-backed fixed income; general fixed income; volatility; multi-strategy.

**Equity strategies**: market neutral; fundamental growth; fundamental value; quantitative directional; short bias.

**Macro strategies**: based on global economic trends.

**Hedge fund fees**:

- "2 and 20": 2% management fee plus 20% incentive fee.
- Hard hurdle rate: incentive fee only on return above hurdle rate.
- Soft hurdle rate: incentive fee on whole return, but only paid if return is greater than hurdle rate.
- High water mark: no incentive fee until value exceeds previous high.

### PRIVATE CAPITAL

**Leveraged buyouts**: management buyouts (existing managers), management buy-ins (new managers)

Venture capital stages of development:

- Formative stage: angel investing, seed stage, early stage.
- Later stage: expand production, increase sales.
- Mezzanine stage: prepare for IPO.

**Exit strategies**: trade sale; IPO; recapitalization; secondary sale; write-off.

### REAL ESTATE

Includes residential property; commercial property; real estate investment trusts (REITs); whole loans; construction loans.

### COMMODITIES

Contango: futures price > spot price.

Backwardation: futures price < spot price.

futures price  $\approx$  spot price(1 + R<sub>e</sub>) + storage costs - convenience yield

### INFRASTRUCTURE

Long-lived assets for public use, including transportation, utility, communications, social.

**Brownfield**: Existing infrastructure.

**Greenfield**: Infrastructure to be built.

## DERIVATIVES

### ARBITRAGE AND REPLICATION

- Law of one price-, two assets with identical cash flows in the future, regardless of future events, should have the same price.
- Two assets with uncertain returns can be combined in a portfolio that will have a certain payoff. If a portfolio has a certain payoff, the portfolio should yield the risk-free rate. For this reason, derivatives values are based on risk-neutral pricing.

### DERIVATIVES VALUES VS. PRICES

The price of a forward, futures, or swap contract is the forward price stated in the contract and is set such that the contract has a value of zero at initiation. Value may change during the contract's life with opposite gains/losses to the long and short.

### FORWARD CONTRACT VALUE

At time  $t_r$ .

$$V(T) = [S + PV(\text{costs}) - PV(\text{benefits})] - CF_0(T) \alpha + R_f \alpha (T-t)$$

### FUTURES VS. FORWARDS

Futures are standardized, exchange-traded forward contracts that require daily cash settlement of mark-to-market gains and losses.

### FORWARD RATE AGREEMENTS (FRA)

Can be viewed as a forward contract to borrow/ lend money at a certain rate at some future date.

### INTEREST RATE SWAPS

May be replicated by a series of FRAs with present values at swap initiation that sum to zero.

### OPTIONS

- Buyer of a call option—long asset exposure.
- Writer (seller) of a call option—short asset exposure.
- Buyer of a put option—short asset exposure.
- Writer (seller) of a put option—long asset exposure.

$$\text{intrinsic value of a call option} = \text{Max}[0, S - X]$$

$$\text{intrinsic value of a put option} = \text{Max}[0, X - S]$$

### AMERICAN VS. EUROPEAN OPTIONS

American options allow the owner to exercise the option any time before or at expiration. European options can be exercised only at expiration.

### FACTORS THAT AFFECT OPTION VALUES

Increase in:	Calls	Puts
Asset price	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility	Increase	Increase
Time to expiration	Increase	Increase*
Holding costs	Increase	Decrease
Holding benefits	Decrease	Increase

\* Except some deep-in-the-money European puts.

### PUT-CALL PARITY

The put-call parity relationship for European options at time  $t$ .

$$c + X(1 + R_f)^{-T} = S + p$$

Each security in the put-call parity relationship can be expressed as:

$$S = c - p + X(1 + R_f)^{-T}$$

$$p = c - S + X(1 + R_f)^{-T}$$

$$c = S + p - X(1 + R_f)^{-T}$$

$$X(1 + R_f)^{-T} = S + p - c$$

### PUT-CALL FORWARD PARITY

The present value of the forward price of the underlying asset,  $F_0(T) (1 + R_f)^{-T}$ , can be substituted for  $S$  in any of the put-call parity relationships at time 0.

## PORTFOLIO MANAGEMENT

### INVESTMENT POLICY STATEMENT

#### Investment objectives:

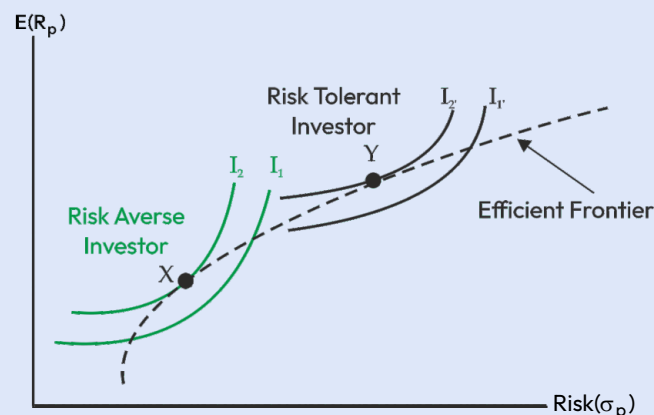
- Return objectives.
- Risk tolerance.

#### Constraints:

- Liquidity needs.
- Time horizon.
- Tax concerns.
- Legal and regulatory factors.
- Unique circumstances.

### COMBINING PREFERENCES WITH THE OPTIMAL SET OF PORTFOLIOS

Markowitz efficient frontier is the set of portfolios that have highest return for given level of risk.

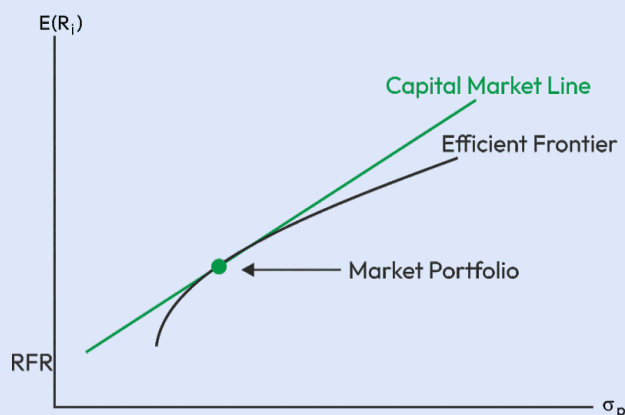


### SECURITY MARKET LINE (SML)

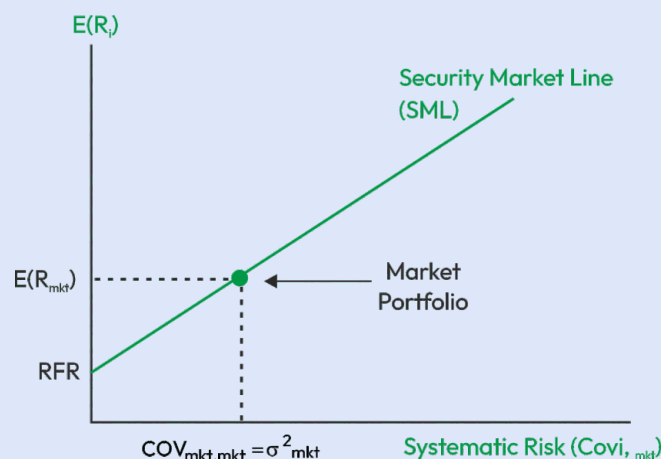
Investors should only be compensated for risk relative to market. **Unsystematic risk** is diversified away; investors are compensated for **systematic risk**. The equation of the SML is the CAPM, which is a return/systematic risk equilibrium

relationship.

**total risk = systematic + unsystematic risk**



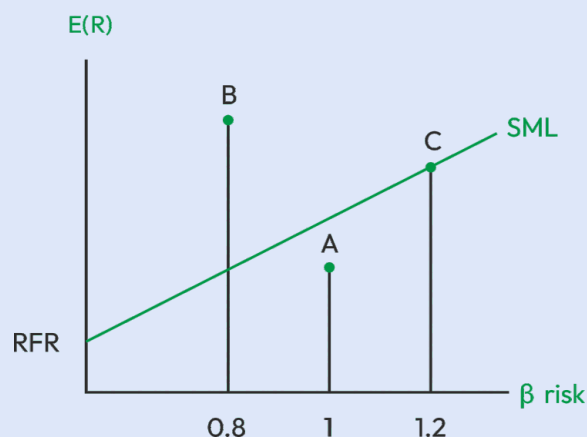
$$\text{CAPM: } E(R_i) = \text{RFR} + \beta_i [E(R_{\text{mkt}}) - \text{RFR}]$$



## THE SML AND EQUILIBRIUM

### Identifying mispriced stocks:

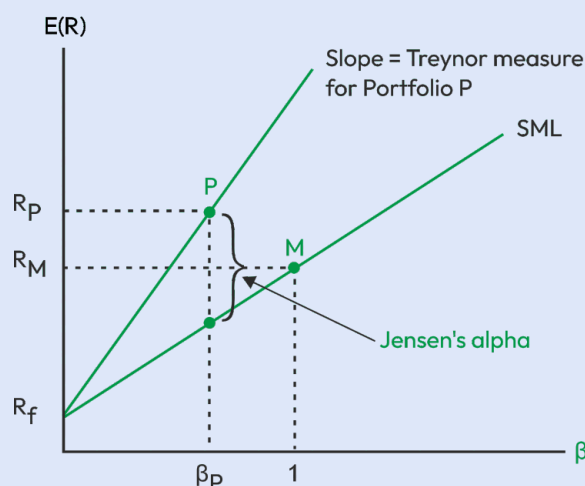
- Consider three stocks (A, B, C) and SML.  
Estimated stock returns should plot on SML.
- A return plot over the line is underpriced.
  - A return plot under the line is overpriced.



## RISK-ADJUSTED RETURNS

**Sharpe ratio** and **M-squared** measure excess return per unit of total risk.

**Treynor measure** and **Jensens alpha** measure excess return per unit of **systematic risk**.



## BEHAVIORAL BIASES

**Cognitive errors, belief perseverance:** Conservatism, confirmation, representativeness, control, hindsight

**Cognitive errors, information processing:** Anchoring and adjustment, mental accounting, framing, availability

**Emotional biases:** Loss aversion, overconfidence, self-control, status quo, endowment, regret aversion

## TECHNICAL ANALYSIS

**Reversal patterns:** head and shoulders, inverse H&S, double/triple top or bottom.

**Continuation patterns:** triangles, rectangles, pennants, flags.

**Price-based indicators:** moving averages, Bollinger bands, momentum oscillators (rate of change, RSI, stochastic, MACD).

**Sentiment indicators:** put/call ratio, VIX, margin debt.

## ETHICAL AND PROFESSIONAL STANDARDS

### STANDARDS OF PROFESSIONAL CONDUCT

#### I. Professionalism

- Knowledge of the Law
- Independence and Objectivity
- Misrepresentation
- Misconduct

#### II. Integrity of Capital Markets

- Material Nonpublic Information
- Market Manipulation

#### III. Duties to Clients

- Loyalty, Prudence, and Care
- Fair Dealing
- Suitability
- Performance Presentation
- Preservation of Confidentiality

#### IV. Duties to Employers

- Loyalty

- B. Additional Compensation Arrangements
- C. Responsibilities of Supervisors

**V. Investment Analysis, Recommendations, and Actions**

- A. Diligence and Reasonable Basis
- B. Communication with Clients and Prospective Clients
- C. Record Retention

**VI. Conflicts of Interest**

- A. Disclosure of Conflicts
- B. Priority of Transactions
- C. Referral Fees

**VII. Responsibilities as a CFA Institute Member or CFA Candidate**

- A. Conduct as Participants in CFA Institute Programs
- B. Reference to CFA Institute, the CFA Designation, and the CFA Program

## GLOBAL INVESTMENT PERFORMANCE STANDARDS

**Definition of firm:** Corporation, subsidiary, or division held out to clients as a business entity. All fee-paying discretionary portfolios must be included in at least one composite.

**Verification:** Optional, but if chosen it must be carried out by an independent third party.

**GIPS standards for firms:**

1. Fundamentals of Compliance
2. Input Data and Calculation Methodology
3. Composite and Pooled Fund Maintenance
4. Composite Time-Weighted Return Report
5. Composite Money-Weighted Return Report
6. Pooled Fund Time-Weighted Return Report
7. Pooled Fund Money-Weighted Return Report
8. GIPS Advertising Guidelines